LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2013**

MT 1818 - DIFFERENTIAL GEOMETRY

Date : 13/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

Answer ALL the Questions:

1. a) Obtain the equation of the tangent at any point u on the circular helix. (5)

OR

b) Prove that the curvature is the rate of change of angle of contingency with respect to the arc length. (5)

c) (i) Show that the necessary and sufficient condition for a curve to be plane curve is $[\bar{r}', \bar{r}'', \bar{r}'''] = 0$. (7)

ii) For the curve $\bar{r} = (a(3u - u^3), 3au^2, a(3u + u^3))$, show that the curvature and torsion are equal. (8)

OR

d) Derive the Serret-Frenet formulae. Express them in terms of Darboux vector. (15)

2. a) Find the plane that has three point contact at the origin with the curve $x = u^4 - 1, y = u^3 - 1, z = u^2 - 1$. (5)

b) Prove that necessary and sufficient condition for a curve to be helix is that the ratio of curvature to torsion is constant. (5)

OR

c) Derive the equation of involute and evolute. (15)

OR

d) State and prove the fundamental theorem of space curves. (15)

3. a) Find the magnitudes of first fundamental form of the sphere: $x = a\sin\theta\cos\varphi$; $y = a\sin\theta\sin\varphi$; $z = a\cos\theta$. (5)

OR

b) Prove that the metric is invariant under a parametric transformation. (5)

c) Prove that the necessary and sufficient condition for the surface may be developable is that its Gaussian surface is zero. (15)

OR

d) Define and derive polar and rectifying developable associated with a space curve. (15)

4. a) Derive the equation satisfying principal curvature at a point on a surface. (5)

OR

b) Find the principal direction at any point on the surface $\bar{r} = (u\cos v, u\sin v, f(u))$. (5)

c) i) Define an umbilic point.

ii) Prove that the sphere is the only surface in which all points are umbilics.

iii) State and prove Euler's theorem. (2 + 6 + 7)

OR

d) (i) Prove that on the general surface a necessary and sufficient condition that the cure $v = c$ be geodesic is $EE_v + FE_u - 2EF_u = 0$.

(ii) Show that the curves $u + v$ = a constant are geodesics on a surface with metric $(1 + u^2)du^2 - 2uvdudv + (1 + v^2)dv^2$. (8 + 7)

5. a) Derive Weingarten equation. (5)

OR

b) State and prove Hilbert's Theorem. (5)

c) Derive Gauss equation. (15)

OR

d) State and prove the fundamental theorem of surface theory and demonstrate. (15)